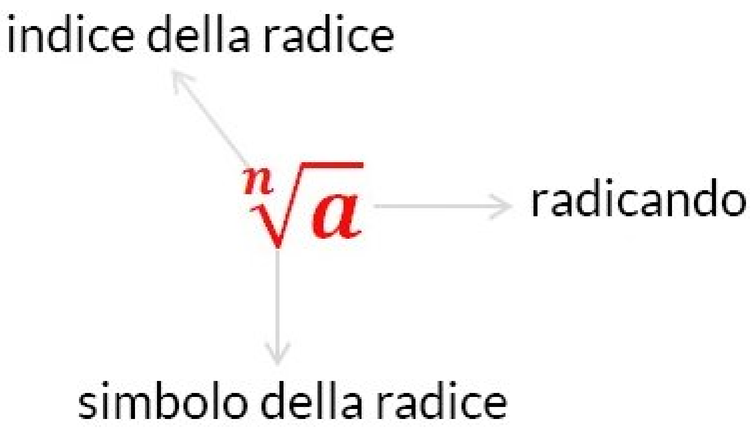
I RADICALI



RACCOLTA DI ESERCIZI CON SOLUZIONE

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Ecco a voi una raccolta di esercizi su radici e radicali. La prima parte è una raccolta di esercizi svolti, mentre la seconda è una raccolta sono esercizi da svolgere con soluzione finale.

BUON LAVORO!

Esercizi Svolti

A. Semplifica le seguenti espressioni con numeri irrazionali:

1. $\sqrt[5]{49}$ è irriducibile perché $\sqrt[5]{49} = \sqrt[5]{7^2}$ e $M.C.D.(5;2) = 1$
2. $\sqrt[6]{49} = \sqrt[3\cdot 2]{7^2} = \sqrt[3]{7}$
3. $\sqrt[12]{a^8} = \sqrt[3]{a^2}$ $\qquad \sqrt[4]{a^8} = a^2$ $\qquad \sqrt[2]{a^4} = a^2$
4. $\sqrt[6]{8} = \sqrt[6]{2^3} = \sqrt[6:3]{2^{3:3}} = \sqrt[2]{2}$ $\qquad \sqrt[9]{-5^{18}} = -\sqrt[9]{5^{18}} = -5^2 = -25$
5. $\sqrt[3]{x^3} = x$ $\qquad \sqrt[10]{a^{15}} = \sqrt[2]{a^3}$, $C.E.:\ a \geq 0$
6. $\sqrt[6]{16} = \sqrt[6]{2^4} = \sqrt[3]{2^2} = \sqrt[3]{4}$ $\qquad \sqrt[6]{8} = \sqrt[6]{2^3} = \sqrt[2]{2}$
7. $\sqrt[6]{(-8)^2} = \sqrt[3]{|-8|} = \sqrt[3]{8} = 2$ $\qquad$ la scrittura $\sqrt[6]{(-8)^2} = \sqrt[3]{-8}$ è errata, perché $\sqrt[6]{(-8)^2} \geq 0$ mentre $\sqrt[3]{-8} < 0$
8. $\sqrt[6]{(-2)^2} = \sqrt[3]{|-2|} = \sqrt[3]{2}$ $\qquad$ la scrittura $\sqrt[6]{(-2)^2} = \sqrt[3]{-2}$ è evidentemente errata.
9. $\sqrt[4]{\left(2-\sqrt{5}\right)^4} = |2-\sqrt{5}| = -(2-\sqrt{5})$ $\qquad$ perchè $2-\sqrt{5} < 0$
10. $\sqrt[6]{8a^3 - 12a^2 + 6a - 1} = \sqrt[6]{(2a-1)^3} = \sqrt[2]{2a-1}$ $\quad$ con $C.E.:\ a \geq \frac{1}{2}$
11. $\sqrt[2]{a^2 - 6a + 9} = \sqrt[2]{(a-3)^2} = |a-3|$
12. $\sqrt[12]{(5a-2)^8} = \sqrt[3]{(5a-2)^2}$
13. $\left(1+\sqrt{2}\right)^2 + \left(1+\sqrt{2}\right)\cdot\sqrt{2} + \left(\sqrt{2}+1\right)\cdot\left(\sqrt{2}-1\right) = 1+2+2\sqrt{2}+\sqrt{2}+2+2-1 = 6+3\sqrt{2}$.
14. $2\sqrt{8} - 3\sqrt{18} + 5\sqrt{12} - \sqrt{200} + \frac{6}{\sqrt{2}} = 2\cdot 2\sqrt{2} - 3\cdot 3\sqrt{2} + 5\cdot 2\sqrt{3} - 10\sqrt{2} + \frac{6}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} =$

   $= 4\sqrt{2} - 9\sqrt{2} + 10\sqrt{3} - 10\sqrt{2} + \dfrac{6\sqrt{2}}{2} = 4\sqrt{2} - 9\sqrt{2} + 10\sqrt{3} - 10\sqrt{2} + 3\sqrt{2} = 10\sqrt{3} - 12\sqrt{2}$.
15. $\left(2\sqrt{3}-5\sqrt{2}\right)^2 - \frac{\sqrt{6}+\sqrt{3}}{2\sqrt{2}+2} - \frac{15}{\sqrt{3}}\cdot\left(\sqrt{3}-4\sqrt{2}\right) = 12 + 50 - 20\sqrt{6} - \frac{\sqrt{6}+\sqrt{3}}{2\sqrt{2}+2}\cdot\frac{2\sqrt{2}-2}{2\sqrt{2}-2} - 15 + 60\frac{\sqrt{2}}{\sqrt{3}} =$

   $= 62 - 20\sqrt{6} - \dfrac{2\sqrt{12} - 2\sqrt{6} + 2\sqrt{6} - 2\sqrt{3}}{8-4} - 15 + 60\dfrac{\sqrt{2}}{\sqrt{3}}\cdot\dfrac{\sqrt{3}}{\sqrt{3}} =$

   $= 62 - 20\sqrt{6} - \dfrac{4\sqrt{3}-2\sqrt{3}}{4} - 15 + 60\dfrac{\sqrt{6}}{3} = 62 - 20\sqrt{6} - \dfrac{2\sqrt{3}}{4} - 15 + 20\sqrt{6} = 47 - \dfrac{\sqrt{3}}{2}$.
16. $\frac{1}{(3+\sqrt{2})(2+\sqrt{3})(3-\sqrt{2})(2-\sqrt{3})} = \frac{1}{(3+\sqrt{2})(3-\sqrt{2})(2+\sqrt{3})(2-\sqrt{3})} = \frac{1}{(9-2)(4-3)} = \frac{1}{7\cdot 1} = \frac{1}{7}$.
17. $\sqrt[5]{2\sqrt[4]{2}} = \sqrt[5]{\sqrt[4]{2^4\cdot 2}} = \sqrt[5]{\sqrt[4]{2^5}} = \sqrt[20]{2^5} = \sqrt[4]{2}$.
18. $\left[\left(5^{\sqrt[3]{2}}\right)^{\sqrt{2}}\right]^{\sqrt[6]{2}} = 5^{\sqrt[3]{2}\cdot\sqrt{2}\cdot\sqrt[6]{2}} = 5^{\sqrt[6]{2^2}\cdot\sqrt[6]{2^3}\cdot\sqrt[6]{2}} = 5^{\sqrt[6]{2^2\cdot 2^3\cdot 2}} = 5^{\sqrt[6]{2^6}} = 5^2 = 25$.
19. $\sqrt{7+\sqrt{9+\sqrt{4+5180}}} = \sqrt{7+\sqrt{9+\sqrt{5184}}} = \sqrt{7+\sqrt{9+72}} = \sqrt{7+\sqrt{81}} = \sqrt{7+9} = \sqrt{16} = 4$.
20. $\left(1-2\sqrt{3}\right)\cdot\left(1+\sqrt{3}\right) - \left(\sqrt{3}-5\right)^2 = 1+\sqrt{3}-2\sqrt{3}-2\cdot 3-\left(3+25-10\sqrt{3}\right) =$

$= 1+\sqrt{3}-2\sqrt{3}-6-3-25+10\sqrt{3} = -33+9\sqrt{3}$.

21. Verifica che: $\left(\sqrt{10}\right)^2+\left(\sqrt{5}+\sqrt{2}\right)^2=\left(\sqrt{17+2\sqrt{10}}\right)^2$

$10+5+2+2\sqrt{10}=17+2\sqrt{10}$; $17+2\sqrt{10}=17+2\sqrt{10}$.

22. $(4\sqrt{3}+\sqrt{2})^2+(\sqrt{3}+3\sqrt{2})\cdot(\sqrt{3}-3\sqrt{2})-15\cdot\frac{\sqrt{3}-3\sqrt{2}}{\sqrt{3}+3\sqrt{2}} = 48+2+8\sqrt{6}+3-18-15\cdot\frac{\sqrt{3}-3\sqrt{2}}{\sqrt{3}+3\sqrt{2}}\cdot\frac{\sqrt{3}-3\sqrt{2}}{\sqrt{3}-3\sqrt{2}} =$

$= 35+8\sqrt{6}-15\cdot\frac{3+18-6\sqrt{6}}{3-18} = 35+8\sqrt{6}-15\cdot\frac{21-6\sqrt{6}}{-15} = 35+8\sqrt{6}+21-6\sqrt{6} = 56+2\sqrt{6}$.

23. $3\sqrt{18}-2\sqrt{8}+5\sqrt{12}-6\sqrt{32}-4\sqrt{48} = 3\cdot3\sqrt{2}-2\cdot2\sqrt{2}+5\cdot2\sqrt{3}-6\cdot4\sqrt{2}-4\cdot4\sqrt{3}=$

$= 9\sqrt{2}-4\sqrt{2}+10\sqrt{3}-24\sqrt{2}-16\sqrt{3} = -19\sqrt{2}-6\sqrt{3}$.

24. $\frac{2}{3+2\sqrt{6}}-\frac{\sqrt{2}}{2\sqrt{3}}+\frac{\sqrt{2}}{2(\sqrt{3}+2\sqrt{2})} = \frac{2}{3+2\sqrt{6}}\cdot\frac{3-2\sqrt{6}}{3-2\sqrt{6}}-\frac{\sqrt{2}}{2\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}+\frac{\sqrt{2}}{2(\sqrt{3}+2\sqrt{2})}\cdot\frac{\sqrt{3}-2\sqrt{2}}{\sqrt{3}-2\sqrt{2}} =$

$= \frac{6-4\sqrt{6}}{9-24}-\frac{\sqrt{6}}{6}+\cdot\frac{\sqrt{6}-4}{2\cdot(3-8)} = \frac{4\sqrt{6}-6}{15}-\frac{\sqrt{6}}{6}+\frac{4-\sqrt{6}}{10} = \frac{8\sqrt{6}-12-5\sqrt{6}+12-3\sqrt{6}}{30} = 0$.

25. $3\sqrt{27}-5\sqrt{32}+2\sqrt{300}-\frac{6}{\sqrt{3}}+\frac{6}{\sqrt{3}-\sqrt{2}} = 3\cdot3\sqrt{3}-5\cdot\sqrt{2^5}+2\sqrt{100}\sqrt{3}-\frac{6}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}+\frac{6}{\sqrt{3}-\sqrt{2}}\cdot\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}} =$

$= 9\sqrt{3}-20\sqrt{2}+20\sqrt{3}-\frac{6\sqrt{3}}{3}+\frac{6\cdot(\sqrt{3}+\sqrt{2})}{3-2} = 9\sqrt{3}-20\sqrt{2}+20\sqrt{3}-2\sqrt{3}+6\sqrt{3}+6\sqrt{2} =$

$= 33\sqrt{3}-14\sqrt{2}$.

26. $\sqrt[3]{(2-\pi)^3} = 2-\pi$.

27. $\sqrt[4]{(2-\pi)^4} = |2-\pi| = -(2-\pi)$ perché $2-\pi<0$.

28. $\sqrt[4]{(\pi-2)^4} = |\pi-2| = +(\pi-2)$ perché $\pi-2>0$.

29. $\sqrt[3]{\left(\sqrt{5}-\sqrt{6}\right)^3} = \sqrt{5}-\sqrt{6}$.

30. $\sqrt[4]{\left(\sqrt{5}-\sqrt{6}\right)^4} = \left|\sqrt{5}-\sqrt{6}\right| = -\left(\sqrt{5}-\sqrt{6}\right)$ perché $\sqrt{5}-\sqrt{6}<0$.

31. $\sqrt[4]{\left(\sqrt{6}-\sqrt{5}\right)^4} = \left|\sqrt{6}-\sqrt{5}\right| = +\left(\sqrt{6}-\sqrt{5}\right)$ perché $\sqrt{6}-\sqrt{5}>0$.

32. $\sqrt{\left(\sqrt{2}-\sqrt{3}\right)^2} = \left|\sqrt{2}-\sqrt{3}\right| = -\left(\sqrt{2}-\sqrt{3}\right)$ perché $\sqrt{2}-\sqrt{3}<0$.

33. $\sqrt{\left(\sqrt{2}-1\right)^2}+\sqrt{\left(\sqrt{2}-\sqrt{3}\right)^2} = \sqrt{2}-1-\left(\sqrt{2}-\sqrt{3}\right) = \sqrt{2}-1-\sqrt{2}+\sqrt{3} = -1+\sqrt{3}$

B. Semplifica le seguenti espressioni letterali irrazionali:

34. $\sqrt{16a^4b^2} = 4a^2|b|$

35. $\sqrt{x^2 - y^2}$ radicale irriducibile

36. $\sqrt[6]{a^3 - 9a^2 + 27a - 27} = \sqrt[6]{(a-3)^3} = \sqrt{a-3}$ con la condizione di esistenza $a \geq 3$

37. $\sqrt[12]{\frac{x^3}{x^3-3x^2+3x-1}} = \sqrt[12]{\frac{x^3}{(x-1)^3}} = \sqrt[4]{\frac{x}{x-1}}$ con le C.E.: $\frac{x^3}{(x-1)^3} \geq 0$ ossia $x < 0 \quad \vee \quad x \geq 1$

38. $\sqrt[4]{a^2 + 4b^2 + 4ab} = \sqrt[4]{(a+b)^2} = \sqrt{|a+b|}$

39. $\sqrt[15]{\frac{x^3+3x^2+3x+1}{x^6}} = \sqrt[15]{\frac{(x+1)^3}{x^6}} = \sqrt[5]{\frac{x+1}{x^2}}$

40. $\left(\sqrt[5]{3-x}\right)^5 = 3 - x$

41. $\left(\sqrt{3-x}\right)^2 = 3 - x$ con $3 - x \geq 0$ ossia $x \leq 3$

42. $\left(\sqrt{3+x}\right)^6 = (3+x)^3$ con $3 + x \geq 0$ ossia $x \geq -3$

43. $\sqrt{4x^2+4x+1} + \sqrt[4]{(x^2-6x+9)^2} = \sqrt{(2x+1)^2} + \sqrt[4]{(x-3)^4} = |2x+1| + |x-3|$.

44. $\sqrt{x+1} + \sqrt{4x+4} + \sqrt{9x+9} = \sqrt{x+1} + \sqrt{4(x+1)} + \sqrt{9(x+1)} = \sqrt{x+1} + 2\sqrt{x+1} + 3\sqrt{x+1} =$
$6\sqrt{x+1}$ $\quad con$ C.E: $x \geq 1$

45. $\sqrt{9x^5 - 18x^4} + \sqrt{4x-8} - 3\sqrt{x^3 - 8 - 6x^2 + 12x} = \sqrt{9x^4(x-2)} + \sqrt{4(x-2)} - 3\sqrt{(x-2)^3} =$ C.E: $x \geq 2$
$= 3x^2\sqrt{x-2} + 2\sqrt{x-2} - 3(x-2)\sqrt{x-2} = (3x^2 + 2 - 3x + 6)\sqrt{x-2} = (3x^2 - 3x + 8)\sqrt{x-2}$.

46. $\frac{1}{\sqrt{a+1}-\sqrt{a}} + \frac{1}{\sqrt{a+1}+\sqrt{a}} - \sqrt[4]{16a^2 + 32a + 16} = \frac{\sqrt{a+1}+\sqrt{a}+\sqrt{a+1}-\sqrt{a}}{(\sqrt{a+1}-\sqrt{a})\cdot(\sqrt{a+1}-\sqrt{a})} - \sqrt[4]{16\cdot(a+1)^2} =$
$= \frac{\sqrt{a+1}+\sqrt{a+1}}{\left(\sqrt{a+1}\right)^2 - \left(\sqrt{a}\right)^2} - 2\sqrt{a+1} = \frac{2\sqrt{a+1}}{a+1-a} - 2\sqrt{a+1} = 2\sqrt{a+1} - 2\sqrt{a+1} = 0$ $\quad con$ C.E: $\begin{cases} a \geq 0 \\ a+1 \geq 0 \end{cases}$ $\quad a \geq 0$

47. $\sqrt{\frac{a^2-1}{a^2+a-2}} : \sqrt[3]{\frac{a^2-4}{a+1}} \cdot \sqrt[6]{\frac{a+2}{a^2+2a+1}} = \sqrt{\frac{(a+1)(a-1)}{(a-1)(a+2)}} : \sqrt[3]{\frac{(a+2)(a-2)}{a+1}} \cdot \sqrt[6]{\frac{a+2}{(a+1)^2}} =$
$= \sqrt{\frac{a+1}{a+2}} : \sqrt[3]{\frac{(a+2)(a-2)}{a+1}} \cdot \sqrt[6]{\frac{a+2}{(a+1)^2}} = \sqrt[6]{\frac{(a+1)^3}{(a+2)^3}} : \sqrt[6]{\frac{(a+2)^2(a-2)^2}{(a+1)^2}} \cdot \sqrt[6]{\frac{a+2}{(a+1)^2}} =$
$= \sqrt[6]{\frac{(a+1)^3}{(a+2)^3} \cdot \frac{(a+1)^2}{(a+2)^2(a-2)^2} \cdot \frac{a+2}{(a+1)^2}} = \sqrt[6]{\frac{(a+1)^3}{(a+2)^4\cdot(a-2)^2}}$
$con\ C.E.: \begin{cases} \frac{(a+1)(a-1)}{(a-1)(a+2)} \geq 0 \\ a+2 \geq 0 \end{cases}$ $\quad \begin{cases} a < 2 \quad \vee \quad -1 < a < 1 \quad \vee \quad a < 1 \\ a \geq -2 \end{cases}$ $\quad -1 < a < 1 \quad \vee \quad a < 1$.

48. $\sqrt[20]{\dfrac{(x^2+2x+1)(x^2+6x+9)}{(16x^2-32x+16)}} = \sqrt[20]{\dfrac{(x+1)^2(x+3)^2}{16(x-1)^2}} = \sqrt[10]{\dfrac{|x+1||x+3|}{4|x-1|}}$.

49. $\sqrt[6]{\dfrac{1+2x^2y}{x^4y^2}+1} = \sqrt[6]{\dfrac{1+2x^2y+x^4y^2}{x^4y^2}} = \sqrt[6]{\dfrac{(1+x^2y)^2}{x^4y^2}} = \sqrt[3]{\dfrac{|1+x^2y|}{x^2|y|}} \qquad x\neq 0 \quad y\neq 0$.

50. $\sqrt[3]{\frac{a}{b}+\frac{b}{a}+2} \cdot \sqrt[4]{\frac{a}{b^2}+\frac{b}{a^2}+\frac{3(a+b)}{ab}} : \sqrt{\frac{a^3+b^3+3a^2b+3ab^2}{a^2b^2}} = \sqrt[3]{\frac{a^2+b^2+2ab}{ab}} \cdot \sqrt[4]{\frac{a^3+b^3+3a^2b+3ab^2}{a^2b^2}} \cdot \sqrt{\frac{a^2b^2}{(a+b)^3}} =$

$= \sqrt[3]{\dfrac{(a+b)^2}{ab}} \cdot \sqrt[4]{\dfrac{(a+b)^3}{a^2b^2}} \cdot \sqrt{\dfrac{a^2b^2}{(a+b)^3}} = \sqrt[12]{\dfrac{(a+b)^8}{a^4b^4}} \cdot \sqrt[12]{\dfrac{(a+b)^9}{a^6b^6}} \cdot \sqrt[12]{\dfrac{a^{12}b^{12}}{(a+b)^{18}}} =$

$= \sqrt[12]{\dfrac{(a+b)^8}{a^4b^4} \cdot \dfrac{(a+b)^9}{a^6b^6} \cdot \dfrac{a^{12}b^{12}}{(a+b)^{18}}} = \sqrt[12]{\dfrac{a^2b^2}{a+b}} \qquad con\ C.E.:\ a+b \geq 0$

51. $\left(\sqrt{\frac{2x-1}{2x+1}}+\sqrt{\frac{1}{4x^2-1}}\right) : \frac{1}{\sqrt{2x-1}} - \frac{2x}{\sqrt{2x+1}} = \left(\sqrt{\frac{2x-1}{2x+1}}+\sqrt{\frac{1}{(2x+1)(2x-1)}}\right) \cdot \sqrt{2x-1} - \frac{2x}{\sqrt{2x+1}} =$

$= \sqrt{\dfrac{2x-1}{2x+1}} \cdot \sqrt{2x-1} + \sqrt{\dfrac{1}{(2x+1)(2x-1)}} \cdot \sqrt{2x-1} - \dfrac{2x}{\sqrt{2x+1}} =$

$= \sqrt{\dfrac{(2x-1)^2}{2x+1}} + \sqrt{\dfrac{2x-1}{(2x+1)(2x-1)}} - \dfrac{2x}{\sqrt{2x+1}} = \dfrac{2x-1}{\sqrt{2x+1}} + \sqrt{\dfrac{1}{2x+1}} - \dfrac{2x}{\sqrt{2x+1}} =$

$= \dfrac{2x-1}{\sqrt{2x+1}} + \dfrac{1}{\sqrt{2x+1}} - \dfrac{2x}{\sqrt{2x+1}} = \dfrac{2x-1+1-2x}{\sqrt{2x+1}} = \dfrac{0}{\sqrt{2x+1}} = 0$.

$con\ C.E.: \quad \begin{cases} \dfrac{2x-1}{2x+1} \geq 0 \\ (2x+1)(2x-1) > 0 \end{cases} \qquad \begin{cases} x < -\dfrac{1}{2} \ \vee \ x \geq +\dfrac{1}{2} \\ x < -\dfrac{1}{2} \ \vee \ x > +\dfrac{1}{2} \end{cases} \qquad x < -\dfrac{1}{2} \ \vee \ x > +\dfrac{1}{2}$

52. $\sqrt{\dfrac{x-2}{x-1}} \cdot \sqrt[3]{\dfrac{x-1}{x-2}} : \sqrt[4]{\dfrac{x-2}{x-1}} = \sqrt[12]{\left(\dfrac{x-2}{x-1}\right)^6} \cdot \sqrt[12]{\left(\dfrac{x-1}{x-2}\right)^4} \cdot \sqrt[12]{\left(\dfrac{x-1}{x-2}\right)^3} = \sqrt[12]{\dfrac{(x-2)^6}{(x-1)^6} \cdot \dfrac{(x-1)^4}{(x-2)^4} \cdot \dfrac{(x-1)^3}{(x-2)^3}} = \sqrt[12]{\dfrac{x-1}{x-2}}$.

$con\ C.E.: \quad \dfrac{x-2}{x-1} \geq 0 \qquad x < 1 \ \vee \ x \geq 22$.

53. $\left(\sqrt{x-1}+\sqrt{y}\right) \cdot \left(\sqrt{x-1}-\sqrt{y}\right) + \left(\sqrt[6]{2-y}\right)^6$

Le condizioni di esistenza sono: $\begin{cases} x-1 \geq 0 \\ y \geq 0 \\ 2-y \geq 0 \end{cases} \qquad \begin{cases} x \geq 1 \\ y \geq 0 \\ y \leq 2 \end{cases}$ ossia: $C.E.:\ x \geq 1 \ \wedge \ 0 \leq y \leq 2$

$\left(\sqrt{x-1}+\sqrt{y}\right) \cdot \left(\sqrt{x-1}-\sqrt{y}\right) + \left(\sqrt[6]{2-y}\right)^6 = x-1-y+2-y = x-2y+1$ con $C.E.:\ x \geq 1 \ \wedge \ 0 \leq y \leq 2$.

54. $\sqrt[3]{x-5} = \begin{cases} +\sqrt[12]{(x-5)^4} & per\ x \geq 5 \\ -\sqrt[12]{(x-5)^4} & per\ x < 5 \end{cases}$

55. $\sqrt[4]{x-5} = \sqrt[12]{(x-5)^3}$ per $x \geq 5$

56. Semplifica la seguente espressione: $\dfrac{\sqrt[5]{a^2 \cdot \sqrt[3]{a^2}}}{\sqrt[6]{a^5} \cdot \sqrt[4]{a^3}}$ con $a \geq 0$ sia utilizzando le operazioni e le proprietà dei radicali, sia trasformandola in una espressione con esponenti frazionari. Verifica poi, l'uguaglianza dei due risultati ottenuti.

Soluzione 1

$\dfrac{\sqrt[5]{a^2 \cdot \sqrt[3]{a^2}}}{\sqrt[6]{a^5} \cdot \sqrt[4]{a^3}} = \dfrac{\sqrt[5]{\sqrt[3]{a^6 \cdot a^2}}}{\sqrt[12]{a^{10}} \cdot \sqrt[12]{a^9}} = \dfrac{\sqrt[5]{\sqrt[3]{a^8}}}{\sqrt[12]{a^{10} \cdot a^9}} = \dfrac{\sqrt[15]{a^8}}{\sqrt[12]{a^{19}}} = \dfrac{\sqrt[60]{a^{32}}}{\sqrt[60]{a^{95}}} = \sqrt[60]{\dfrac{a^{32}}{a^{95}}} = \sqrt[60]{\dfrac{1}{a^{63}}} =$

$= \dfrac{1}{\sqrt[60]{a^{63}}} = \dfrac{1}{a\sqrt[60]{a^3}} = \dfrac{1}{a\sqrt[20]{a}}$.

Soluzione 2

$$\frac{\sqrt[5]{a^2\cdot\sqrt[3]{a^2}}}{\sqrt[6]{a^5}\cdot\sqrt[4]{a^3}} = \frac{\sqrt[5]{a^2\cdot a^{\frac{2}{3}}}}{a^{\frac{5}{6}}\cdot a^{\frac{3}{4}}} = \frac{\left(a^{\frac{8}{3}}\right)^{\frac{1}{5}}}{a^{\frac{5}{6}+\frac{3}{4}}} = \frac{a^{\frac{8}{15}}}{a^{\frac{19}{12}}} = a^{\frac{8}{15}-\frac{19}{12}} = a^{\frac{32-95}{60}} = a^{-\frac{63}{60}} = a^{-\frac{21}{20}}\ .$$

Soluzione 3

$$\frac{1}{a\sqrt[20]{a}} = \frac{1}{a\cdot a^{\frac{1}{20}}} = \frac{1}{a^{\frac{21}{20}}} = a^{-\frac{21}{20}}\ .$$

C. Trasporta, se possibile, uno o più fattori fuori dal segno di radice:

57. $\sqrt[4]{32x^3y^6z^8} = \sqrt[4]{2^5x^3y^6z^8} = 2|y|z^2\sqrt[4]{2x^3y^2}$ con la condizione di esistenza $x \geq 0$.

58. $\sqrt{x^3-8y^3-6x^2y+12xy^2} = \sqrt{(x-2y)^3} = (x-2y)\sqrt{x-2y}$ con la condizione di esistenza $x-2y \geq 0$.

D. Razionalizza i denominatori delle seguenti frazioni :

59. $\frac{5}{2\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}} = \frac{5\sqrt{5}}{2\sqrt{5}\cdot\sqrt{5}} = \frac{5\sqrt{5}}{2\cdot5} = \frac{\sqrt{5}}{2}$

60. $\frac{6}{\sqrt[7]{8}} = \frac{6}{\sqrt[7]{2^3}}\cdot\frac{\sqrt[7]{2^4}}{\sqrt[7]{2^4}} = \frac{6\sqrt[7]{2^4}}{\sqrt[7]{2^7}} = \frac{6\sqrt[7]{16}}{2} = 3\sqrt[7]{16}$

61. $\frac{5}{\sqrt{7}-\sqrt{2}} = \frac{5}{\sqrt{7}-\sqrt{2}}\cdot\frac{\sqrt{7}+\sqrt{2}}{\sqrt{7}+\sqrt{2}} = \frac{5\cdot\left(\sqrt{7}+\sqrt{2}\right)}{7-2} = \frac{5\cdot\left(\sqrt{7}+\sqrt{2}\right)}{5} = \sqrt{7}+\sqrt{2}$

62. $\frac{x-5}{\sqrt[3]{x}-\sqrt[3]{5}} = \frac{x-5}{\sqrt[3]{x}-\sqrt[3]{5}}\cdot\frac{\sqrt[3]{x^2}+\sqrt[3]{5x}+\sqrt[3]{5^2}}{\sqrt[3]{x^2}+\sqrt[3]{5x}+\sqrt[3]{5^2}} = \frac{(x-5)\cdot\left(\sqrt[3]{x^2}+\sqrt[3]{5x}+\sqrt[3]{5^2}\right)}{\left(\sqrt[3]{x}\right)^3-\left(\sqrt[3]{5}\right)^3} = \frac{(x-5)\cdot\left(\sqrt[3]{x^2}+\sqrt[3]{5x}+\sqrt[3]{5^2}\right)}{x-5} =$

$= \sqrt[3]{x^2}+\sqrt[3]{5x}+\sqrt[3]{25}$.

E. Trasforma i seguenti radicali doppi in somme di radicali semplici:

63. $\sqrt{7+2\sqrt{6}} = \sqrt{7+\sqrt{4\cdot6}} = \sqrt{7+\sqrt{24}}$

Essendo $a^2-b = 7^2-24 = 25$ un quadrato perfetto conviene sviluppare il radicale con la formula:

$$\sqrt{a+\sqrt{b}} = \sqrt{\frac{a+\sqrt{a^2-b}}{2}}+\sqrt{\frac{a-\sqrt{a^2-b}}{2}}$$

$$\sqrt{7+\sqrt{24}} = \sqrt{\frac{7+\sqrt{25}}{2}}+\sqrt{\frac{7-\sqrt{25}}{2}} = \sqrt{\frac{7+5}{2}}+\sqrt{\frac{7-5}{2}} = \sqrt{6}+\sqrt{\frac{2}{2}} = \sqrt{6}+1$$

F. Risolvi il seguente sistema lineare:

64. $\begin{cases} \sqrt{2}x + y - 2 = \sqrt{2} \\ x - \frac{y}{2} + 1 = 0 \end{cases}$ $\begin{cases} \sqrt{2}x + y - 2 = \sqrt{2} \\ 2x - y + 2 = 0 \end{cases}$ $\begin{cases} \sqrt{2}x + y - 2 = \sqrt{2} \\ y = 2x + 2 \end{cases}$ $\begin{cases} \sqrt{2}x + 2x + 2 - 2 = \sqrt{2} \\ - \end{cases}$

65. $\begin{cases} (\sqrt{2} + 2)x = \sqrt{2} \\ - \end{cases}$ $\begin{cases} x = \frac{\sqrt{2}}{\sqrt{2}+2} = \frac{\sqrt{2}}{\sqrt{2}+2} \cdot \frac{\sqrt{2}-2}{\sqrt{2}-2} = \frac{2-2\sqrt{2}}{2-4} = \frac{2(1-\sqrt{2})}{-2} = \sqrt{2} - 1 \\ y = 2 \cdot (\sqrt{2} - 1) + 2 = 2\sqrt{2} - 2 + 2 = 2\sqrt{2} \end{cases}$ $\begin{cases} x = \sqrt{2} - 1 \\ y = 2\sqrt{2} \end{cases}$

Esercizi da svolgere

SEMPLIFICARE, SE POSSIBILE

1) $\sqrt[8]{7^2}\ ;\ \sqrt[10]{4^5}\ ;\ \sqrt[12]{3^4}$ $[\sqrt[4]{7};2;\ \sqrt[3]{3}]$

2) $\sqrt[18]{81}\ ;\ \sqrt[6]{16}\ ;\ \sqrt[16]{256}$ $[\sqrt[9]{9}\ ;\ \sqrt[3]{4}\ ;\ \sqrt{2}]$

3) $\sqrt[9]{27}\ ;\ -\sqrt[8]{25}\ ;\ \sqrt[6]{1000}$ $[\sqrt[3]{3}\ ;\ -\sqrt[4]{5}\ ;\ \sqrt{10}]$

4) $\sqrt[6]{8a^9b^6}\ ;\ \sqrt[5]{32x^{10}y^5}\ ;\ \sqrt[3]{216x^{12}a^6}$ $[\sqrt{2a^3b^2}\ ;2x^2y;6x^4a^2]$

5) $\sqrt[12]{343a^9b^{12}}\ ;\ \sqrt{a^4b^2}\ ;\ \sqrt[4]{16a^8x^{12}}$ $[\sqrt[4]{7a^3b^4}\ ;\ a^2|b|\ ;\ 2a^2|x|^3]$

6) $\sqrt[6]{a^3+3a^2+3a+1}\ ;\ \sqrt{x^2-2x+1}$ $[\sqrt{a+1}\ ;\ |x-1|]$

7) $\sqrt[18]{\left(a^3-6a^2b+12ab^2-8b^3\right)^2}\ ;\ \sqrt[4]{x^4+4x^3+4x^2}$ $[\sqrt[3]{|a-2b|}\ ;\ \sqrt{|x(x+2)|}]$

8) $\sqrt{a^2+b^2}\ ;\ \sqrt{a^2+b^2+2ab}$ $[\sqrt{a^2+b^2}\ ;\ |a+b|]$

9) $\sqrt{x^2+1}\ ;\ \sqrt{x^2-1}$ $[\sqrt{x^2+1}\ ;\ \sqrt{x^2-1}]$

PRODOTTI E QUOZIENTI

10) $\sqrt{3}\cdot\sqrt{27}\ ;\ \sqrt{5}\cdot\sqrt{6}$ $[9;\ \sqrt{30}]$

11) $\left(\sqrt{5}\cdot\sqrt{6}\right)^2\ ;\ \sqrt{9}\cdot\sqrt{16}$ $[30;12]$

12) $\sqrt{3}\cdot\sqrt{12}\cdot\sqrt{4}\ ;\ \sqrt[3]{75}\cdot\sqrt[3]{5}\cdot\sqrt[3]{9}$ $[12;15]$

13) $\sqrt[4]{5}\cdot\sqrt[4]{7}\cdot\sqrt[4]{10}\ ;\ \sqrt[4]{5}\cdot\sqrt[4]{10}\cdot\sqrt[4]{2}$ $[\sqrt[4]{350}\ ;\ \sqrt{10}]$

14) $\sqrt[6]{ab^2}\cdot\sqrt[6]{ab^3}\cdot\sqrt[6]{ab}\ ;\ \sqrt[3]{a^2b}\cdot\sqrt[3]{ab^2}$ $[\sqrt{|a|b^2}\ ;\ ab]$

15) $\sqrt{3}\cdot\sqrt[3]{3}\ ;\ \sqrt{a}\cdot\sqrt[3]{a^4}\cdot\sqrt[4]{a}$ $[\sqrt[6]{3^5}\ ;\ a^2\sqrt[12]{a}]$

16) $\sqrt[5]{x}\cdot\sqrt[10]{x^3}\ ;\ \sqrt{2a}\cdot\sqrt[3]{8a^2}\cdot\sqrt[6]{8a^5}$ $[\sqrt{x}\ ;4a^2]$

17) $\sqrt{\dfrac{a-1}{a}}:\sqrt[3]{\dfrac{a^2-2a+1}{a^2}}$ $\left[\sqrt[6]{\dfrac{a}{a-1}}\right]$

18) $\sqrt[4]{\dfrac{a+1}{x}}\cdot\sqrt[3]{\dfrac{a+1}{x^2}}:\sqrt[12]{\dfrac{x}{(a+1)^5}}$ $\left[\left|\dfrac{a+1}{x}\right|\right]$

19) $\sqrt[7]{\dfrac{x}{2a-1}}:\sqrt[3]{\dfrac{x}{4a^2-4a+1}}\cdot\sqrt[21]{(2a-1)^3x^4}$ $\left[\sqrt[3]{(2a-1)^2}\right]$

20) $\sqrt{\dfrac{(a-b)^2}{ab}}\cdot\sqrt[4]{\dfrac{a}{a^2-b^2}}:\sqrt[8]{\dfrac{b}{a+b}}$ $\left[\sqrt[8]{\dfrac{(a-b)^6}{a^2b^5(a+b)}}\right]$

PORTARE FUORI

21) $\sqrt{128}\ ;\ \sqrt[3]{243}$ $[8\sqrt{2}\ ;\ 3\sqrt[3]{9}]$

22) $\sqrt[4]{a^4b^8c}\ ;\ \sqrt[5]{a^2b^3c^5}$ $[|a|b^2\sqrt[4]{c}\ ;\ c\sqrt[5]{a^2b^3}]$

23) $\sqrt{150}\ ;\ \sqrt{24}$ $[5\sqrt{6}\ ;\ 2\sqrt{6}]$

24) $\sqrt{9xy^4}\ ;\ \sqrt{x^2y^4}$ $[3y^2\sqrt{x}\ ;\ |x|y^2]$

25) $\sqrt{x^3+x^2}\ ;\ \sqrt[4]{x^8+x^4}$ $[|x|\sqrt{x+1}\ ;\ |x|\sqrt[4]{x^4+1}]$

26) $\sqrt{9x^2-9x^4}\ ;\ \sqrt[3]{27x^3+27x^6}$ $[3|x|\sqrt{1-x^2}\ ;\ 3x\sqrt[3]{1+x^3}]$

27) $\sqrt{32a^5b^{12}c^{43}}\ ;\ \sqrt[11]{a^7b^{11}c^{18}d^{29}x^{141}}$ $[4a^2b^6c^{21}\sqrt{2ac}\ ;\ bcd^2x^{12}\sqrt[11]{a^7c^7d^7x^9}]$

PORTARE DENTRO

28) $-2\sqrt{3}\ ;\ \dfrac{1}{3}\sqrt{3}$ $\left[-\sqrt{12}\ ;\ \sqrt{\dfrac{1}{3}}\right]$

29) $\dfrac{2}{3}\sqrt{12}\ ;\ \dfrac{3}{4}\sqrt{18}$ $\left[\sqrt{\dfrac{16}{3}}\ ;\ \sqrt{\dfrac{81}{8}}\right]$

30) $\dfrac{1}{2}\sqrt[3]{2}\ ;\ 5\sqrt[5]{\dfrac{2}{25}}$ $\left[\sqrt[3]{\dfrac{1}{4}}\ ;\ \sqrt[5]{250}\right]$

31) $\dfrac{1}{a-b}\sqrt{b-a}\ ;\ \dfrac{1}{a+b}\sqrt{b+a}$ $\left[-\sqrt{\dfrac{1}{b-a}}\ ;\ \sqrt{\dfrac{1}{b+a}}\right]$

SOMMA, DIFFERENZA, ESPRESSIONI

32) $3\sqrt{3}-5\sqrt{3}+6\sqrt{3}$ $[4\sqrt{3}]$

33) $5\sqrt{2}-8\sqrt{2}+3\sqrt{2}$ $[0]$

34) $\sqrt{5}-\sqrt{3}+2\sqrt{5}+2\sqrt{3}-3\sqrt{5}-3\sqrt{3}$ [$-2\sqrt{3}$]

35) $3\sqrt{18}-6\sqrt{2}+\sqrt{50}$ [$8\sqrt{2}$]

36) $2\sqrt{125}-\sqrt{45}-3\sqrt{20}$ [$\sqrt{5}$]

37) $\sqrt{27}+4\sqrt{12}-\sqrt{48}$ [$7\sqrt{3}$]

38) $3\sqrt{7}-2\sqrt{5}-\sqrt{343}-\sqrt{80}$ [$-4\sqrt{7}-6\sqrt{5}$]

39) $\sqrt{18}+\sqrt{8}-\sqrt{11}-\sqrt{32}-\sqrt{44}+\sqrt{72}$ [$7\sqrt{2}-3\sqrt{11}$]

40) $\left(2\sqrt{5}-1\right)^2-\left(\sqrt{5}-1\right)\cdot\left(\sqrt{5}+1\right)+4\sqrt{5}$ [17]

41) $\left(\sqrt{3}-\sqrt{2}\right)^2+\left(\sqrt{3}+\sqrt{2}\right)^2$ [10]

42) $\left(2\sqrt{7}-1\right)^2+\left(2\sqrt{7}+3\right)^2+\left(2-\sqrt{7}\right)\sqrt{7}$ [$59+10\sqrt{7}$]

43) $\left(3\sqrt{2}+1\right)\left(2\sqrt{2}-3\right)-\left(\sqrt{2}+1\right)\left(2-3\sqrt{2}\right)$ [$13-7\sqrt{2}$]

44) $\left(2\sqrt{5}-\sqrt{3}\right)^2-\left(2\sqrt{5}+\sqrt{3}\right)^2\left(\sqrt{5}-\sqrt{3}\right)-\sqrt{3}\left(\sqrt{5}+2\sqrt{3}\right)$ [$17-5\sqrt{15}-11\sqrt{5}+3\sqrt{3}$]

45) $\sqrt{\left(-2\sqrt{6}+6\sqrt{2}\right)\left(6\sqrt{2}+2\sqrt{6}\right)}-2\sqrt{2}\left(\sqrt{6}-\sqrt{2}\right)$ [4]

46) $\left(3\sqrt{5}-1\right)\left(2\sqrt{5}+3\right)-\left(\sqrt{5}-1\right)^2-\left(2-3\sqrt{5}\right)^2$ [$21\sqrt{5}-28$]

RADICE DI RADICE

47) $\sqrt[3]{\sqrt{7}}$; $\sqrt[4]{\sqrt[3]{16}}$ [$\sqrt[6]{7}$; $\sqrt[3]{2}$]

48) $\sqrt[3]{\sqrt[3]{a^6}}$; $\sqrt[7]{\sqrt[2]{b^{18}}}$ [$\sqrt[3]{a^2}$; $\sqrt[7]{b^9}$]

49) $\sqrt{3\sqrt{2}}$; $\sqrt{2\sqrt[3]{3}}$ [$\sqrt[4]{18}$; $\sqrt[6]{24}$]

50) $\sqrt[3]{4\sqrt{5}}$; $\sqrt[6]{2\sqrt{18}}$ [$\sqrt[6]{80}$; $\sqrt[12]{72}$]

RAZIONALIZZAZIONI

51) $\frac{3}{\sqrt{7}}$; $\frac{5}{2\sqrt{3}}$; $\frac{3}{2\sqrt{2}}$ [$\frac{3\sqrt{7}}{7}$; $\frac{5\sqrt{3}}{6}$; $\frac{3\sqrt{2}}{4}$]

52) $\frac{4}{\sqrt{3}}$; $\frac{1}{2\sqrt{5}}$; $\frac{6}{3\sqrt{2}}$ [$\frac{4\sqrt{3}}{3}$; $\frac{\sqrt{5}}{10}$; $\sqrt{2}$]

53) $\frac{a}{\sqrt{a}}$; $\frac{2}{\sqrt{2}}$; $\frac{5}{\sqrt{5}}$ [$\sqrt{a}$; $\sqrt{2}$; $\sqrt{5}$]

54) $\frac{\sqrt{3}-3\sqrt{2}}{\sqrt{2}}$; $\frac{2\sqrt{3}-1}{\sqrt{3}}$ [$\frac{\sqrt{6}-6}{2}$; $\frac{6-\sqrt{3}}{3}$]

55) $\frac{6\sqrt{3}-3}{\sqrt{3}}$; $\frac{5-\sqrt{5}}{\sqrt{5}}$ [$6-\sqrt{3}$; $\sqrt{5}-1$]

56) $\frac{2}{\sqrt[3]{2}}$; $\frac{3}{\sqrt[3]{3}}$; $\frac{3}{\sqrt[3]{5}}$ [$\sqrt[3]{4}$; $\sqrt[3]{9}$; $\frac{3}{5}\sqrt[3]{25}$]

57) $\frac{20}{\sqrt[3]{100}}$; $\frac{xy}{\sqrt[3]{xy}}$; $\frac{1}{\sqrt[4]{8}}$ [$2\sqrt[3]{10}$; $\sqrt[3]{x^2y^2}$; $\frac{\sqrt[4]{2}}{2}$]

58) $\frac{3}{\sqrt[4]{9}}$; $\frac{5}{\sqrt[5]{25}}$; $\frac{7}{2\sqrt[3]{4}}$ [$\sqrt{3}$; $\sqrt[5]{125}$; $\frac{7}{4}\sqrt[3]{2}$]

59) $\frac{1}{\sqrt[3]{ab^2}}$; $\frac{xy^2}{\sqrt[4]{xy^3}}$; $\frac{8}{\sqrt[3]{6}}$ [$\frac{\sqrt[3]{a^2b}}{ab}$; $y\sqrt[4]{x^3y}$; $\frac{4}{3}\sqrt[3]{36}$]

60) $\frac{4}{\sqrt[3]{16}}$; $\frac{3}{2\sqrt[3]{48}}$; $\frac{2}{\sqrt[4]{32}}$ [$\sqrt[3]{4}$; $\frac{\sqrt[3]{36}}{8}$; $\frac{\sqrt[4]{8}}{2}$]

61) $\frac{1}{\sqrt{3}-\sqrt{2}}$; $\frac{1}{\sqrt{5}-2}$; $\frac{1}{\sqrt{5}+\sqrt{3}}$ [$\sqrt{3}+\sqrt{2}$; $\sqrt{5}+2$; $\frac{\sqrt{5}-\sqrt{3}}{2}$]

62) $\frac{38}{2\sqrt{5}+1}$; $\frac{14}{3\sqrt{2}-2}$; $\frac{6}{3-\sqrt{6}}$ [$2\left(2\sqrt{5}-1\right)$; $3\sqrt{2}+2$; $2\left(3+\sqrt{6}\right)$]

63) $\frac{2+\sqrt{3}}{2-\sqrt{3}}$; $\frac{3\sqrt{5}-1}{3\sqrt{5}+1}$; $\frac{2}{\sqrt{5}-3}$ [$7+4\sqrt{3}$; $\frac{23-3\sqrt{5}}{22}$; $-\frac{\sqrt{5}+3}{2}$]

64) $\frac{a-1}{\sqrt{a}-1}$; $\frac{a-b}{\sqrt{b}-\sqrt{a}}$; $\frac{x}{\sqrt{x}+1}$ [$\sqrt{a}+1$; $-\sqrt{b}-\sqrt{a}$; $\frac{x\left(\sqrt{x}-1\right)}{x-1}$]

RADICALI DOPPI

65) $\sqrt{4-2\sqrt{3}}$; $\sqrt{4-\sqrt{7}}$ [$\sqrt{3}-1$; $\frac{\sqrt{14}-\sqrt{2}}{2}$]

66) $\sqrt{9-\sqrt{17}}$; $\sqrt{4+2\sqrt{3}}$ [$\frac{\sqrt{34}-\sqrt{2}}{2}$; $\sqrt{3}+1$]

67) $\sqrt{10+\sqrt{51}}$; $\sqrt{11-2\sqrt{10}}$ [$\frac{\sqrt{34}+\sqrt{6}}{2}$; $\sqrt{10}-1$]

68) $\sqrt{6-2\sqrt{5}}$; $\sqrt{8-\sqrt{15}}$ [$\sqrt{5}-1$; $\frac{\sqrt{30}-\sqrt{2}}{2}$]

POTENZE A ESPONENTE RAZIONALE

69) $49^{\frac{3}{2}}$; $81^{-\frac{3}{4}}$ [343; $\frac{1}{27}$]

70) $\left(\frac{25}{4}\right)^{-\frac{1}{2}}$; $\left(\frac{8}{125}\right)^{-\frac{2}{3}}$ [$\frac{2}{5}$; $\frac{25}{4}$]

71) $\left(\frac{a^{-2}}{a^{\frac{2}{3}}}\right)^{\frac{1}{2}}$; $\left(\frac{x^{\frac{3}{2}}\cdot x}{x^{-2}}\right)^{-\frac{2}{3}}\cdot x$ [$\frac{\sqrt[3]{a^2}}{a^2}$; $\frac{1}{x^2}$]

72) $\left(\frac{a^2 : a^{-3}}{a^{\frac{1}{2}}}\right)^{\frac{2}{3}}\cdot a^{-1}$; $\left(\frac{a^3\cdot a^{-\frac{1}{2}}}{a^2}\right)^{-\frac{1}{3}} : a^{-2}$ [a^2; $a\sqrt[6]{a^5}$]